

Group - B (Operations Research)

1) Define Convex set.

Prove that the intersection of any finite number of convex sets is also a convex set.

2) Define Convex Combination

Prove that the set of all convex combinations of a finite number of points x_1, x_2, \dots, x_n is a convex set.

3) Solve Graphically the following L.P. Problems.

i) $\min. Z = 2x_1 + 3x_2$

S.t.

$$x_1 + x_2 \leq 4$$

$$6x_1 + 2x_2 \geq 8$$

$$x_1 + 5x_2 \geq 4$$

$$x_1 \leq 3$$

$$x_2 \leq 3$$

and $x_1, x_2 \geq 0$

ii) $\min. Z = 2x_1 + 3x_2$

S.t.

$$-x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \geq 9$$

and $x_1, x_2 \geq 0$

iii) $\max. Z = 2x_1 + x_2$

S.t.

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1 - 2x_2 \leq 1$$

and $x_1, x_2 \geq 0$

4) Prove that Dual of the dual of a given primal, is the primal itself.

5) Write the dual of the following LPP

i) max. $Z = 2x_1 + 3x_2 + x_3$

s.t.

$$4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

ii) min. $Z = x_1 + x_2 + x_3$

s.t.

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$$x_1, x_2 \geq 0, \quad x_3 \text{ is unrestricted in sign}$$

iii) min. $Z = 2x_1 + 5x_2$

s.t.

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4$$

$$\text{and } x_1, x_2, x_3 \geq 0.$$

6) Write the difference between Primal and dual Problems.

7) Solve the following problems by Simplex Method.

i) $\max. z = 3x_1 + 4x_2$

s.t.

$$x_1 + 3x_2 \leq 9$$

$$2x_1 - x_2 \leq 8$$

$$x_1 + x_2 \leq 5$$

and $x_1, x_2 \geq 0$

ii) $\max. z = 2x_1 + 4x_2$

s.t.

$$2x_1 + 3x_2 \leq 48$$

$$x_1 + 3x_2 \leq 42$$

$$x_1 + x_2 \leq 21$$

and $x_1, x_2 \geq 0$.

iii) $\max. z = 2x_1 + 4x_2 + x_3$

s.t.

$$x_1 + 2x_2 \leq 4$$

$$2x_1 + x_2 \leq 3$$

$$x_2 + x_3 \leq 3$$

and $x_1, x_2, x_3 \geq 0$

8) Solve the following L.P.P. by ^{Big-M} Simplex Method (Big-M method)

i) $\min z = 2x_1 + x_2$

s.t.

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

and $x_1, x_2 \geq 0$.

ii) $\max z = x_1 - x_2 + 3x_3$

s.t.

$$x_1 + x_2 \leq 20$$

$$x_1 + x_3 = 5$$

$$x_2 + x_3 \geq 10$$

and $x_1, x_2, x_3 \geq 0$

9) Find the dual of the problem

$$\max. z = 2x_1 - x_2$$

s.t.

$$x_1 + x_2 \leq 10$$

$$-2x_1 + x_2 \leq 2$$

$$4x_1 + 3x_2 \geq 12$$

and $x_1, x_2 \geq 0$

Solve the primal problem by Simplex Method and deduce from the solution to the dual problem.

10) Solve the following minimal assignment problem

Man → Job ↓	1	2	3	4
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	25	30	28	14

11) Solve the assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

12) Solve the following minimal assignment problem

Machin →	1	2	3	4	5
Job ↓					
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

13) Use the Hungarian method to find which of the two jobs should be left undone when each of the four persons will do only one job in the following cost minimizing assignment problem

		Jobs					
		J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
Persons	P ₁	10	9	11	12	8	5
	P ₂	12	10	9	11	9	4
	P ₃	8	11	10	7	12	6
	P ₄	10	7	8	10	10	5